CEBAF-PR-91-005
CEBAF-TH-91-01
WM-91-101
CALT-68-1704

Spectroscopy with Heavy Quark Symmetry *

Nathan Isgur†
CEBAF, Newport News, VA 23606

Mark B. Wise
California Institute of Technology, Pasadena, CA 91125

ABSTRACT

It has recently been shown that hadrons containing a single heavy quark exhibit a new flavor-spin symmetry of QCD. We discuss the implications of this symmetry for the masses and strong decay widths of such hadrons.

* This work was supported by the U.S. Department of Energy under Contracts DE-AC0381-ER40050 and DE-AC05-84ER40150 and by the Natural Sciences and Engineering Research Council of Canada.

† Governor's Distinguished CEBAF Professor at the College of William and Mary, Williamsburg, VA 23185.

The c, b, and t quarks have masses which are much greater than the QCD scale Λ_{QCD}. The physics of hadrons containing one of these quarks is consequently greatly simplified by going over to an effective theory constructed by taking the limit of QCD where the masses of such heavy quarks go to infinity[1-11] with their four-velocities fixed. In this limit, new symmetries of the strong interactions appear which can be used to predict many properties of such hadrons. For N heavy quarks $Q_i, i = 1, ..N$, this effective theory has an SU(2N) flavor-spin symmetry that arises because in the effective theory the couplings of heavy quarks are independent of their mass and spin; for finite $m_{Q_i} \gg \Lambda_{QCD}$, these symmetries become approximate ones which are broken by effects of order Λ_{QCD}/m_{Q_i} and $\alpha_s(m_{Q_i})/\pi$. The consequences of these symmetries for the weak semileptonic decays of mesons and baryons containing a single heavy quark have been worked out[1,12-14]. These results are expected to play an important role in determining the V_{ub} and V_{cb} elements of the Cabibbo-Kobayashi-Maskawa matrix.

In this Letter we obtain some of the consequences of this symmetry for heavy quark spectroscopy, and in particular for strong decay widths. We show that heavy quark symmetry implies *1)* that all of the mass splittings and partial decay widths of the states of a hadronic system containing a single heavy quark are independent of heavy quark flavor, *2)* that each spectroscopic line consists of two degenerate states with total spins $s_\pm = s_\ell \pm \frac{1}{2}$ (apart from possible single lines with $s = \frac{1}{2}$), *3)* that the sum of the partial widths for a strong transition from each state of a doublet $s_\pm$ to the states $s'_\pm$ of another doublet is the same, and *4)* that the ratio of the partial widths for each partial wave amplitude (to be defined below) contributing to the four possible transitions between the doublet $s_\pm$ and the doublet $s'_\pm$ is predicted by the symmetry. We also present simple formulas for these ratios for excited meson decay to the ground state meson multiplet and for excited baryon decay to both the Λ_Q and to the Σ_Q, Σ^*_Q multiplets.

In the limit $m_Q \to \infty$ the spin of the heavy quark $\vec{S}_Q$ and the spin of the light degrees of freedom $\vec{S}_\ell \equiv \vec{S} - \vec{S}_Q$ are separately conserved by the strong interactions; hadrons containing a single heavy quark can therefore be simultaneously assigned the quantum numbers $s_Q, m_Q, \pi_Q, s_\ell, m_\ell$, and π_ℓ, where $\pi_Q = +$ and π_ℓ are the parities of the heavy quark and of the light degrees of freedom. Since the dynamics of the light degrees of freedom are independent of the mass of the heavy quark Q, s_Q, and π_Q, it is convenient in the heavy quark limit to classify states by s_ℓ and π_ℓ. Then, associated with each such state of the light degrees of freedom (with generic quantum numbers n_ℓ) will be a degenerate doublet of hadrons with $s_\pm^\pi = (s_\ell \pm \frac{1}{2})^{\pi_\ell}$ built on each flavor of heavy quark Q (unless $s_\ell = 0$, in which case a single $s = \frac{1}{2}$ state is obtained). Since the light quark state is independent of Q, the spectrum is identical for each flavor up to an overall constant mass shift associated with the heavy quark. Experimental information on these spectral symmetries is for the moment very limited. We now know that the $D^* - D$ splitting[15)] of ~ 145 MeV is reduced to ~ 50 MeV in the $B^* - B$ multiplet[16)], which is certainly consistent with the expected $1/m_Q$ approach to the heavy quark limit[6)]. Unfortunately, we have as yet little information on excited charmed hadron spectroscopy; nevertheless, as we will see below, the information available is also consistent with the expected doublet degeneracy. There is at this moment no data on excited systems containing a b quark, so the predicted equality in the splittings between flavor sectors cannot be compared with experiment.

Since the heavy quark flavor symmetry applies to the complete set of n-point functions of the theory, not only mass splittings, but also all strong decay amplitudes arising from the emission of light quanta like π, η, ρ, $\pi\pi$, ... are independent of heavy quark flavor. For a given flavor, because the spin $\vec{S}_Q$ generates a symmetry, the two states with spins $s_\pm$ must have the same total widths. This equality between the total widths typically arises in a non-trivial way: the two states of a given multiplet can decay (actually or virtually) to both states of every available multiplet with distinct partial widths (or their virtual analogs) whose sum must be identical. The spin symmetry predicts the ratios of these partial widths.

Consider the decays $H_Q \to [H'_Q h]_{LJ_h}$, where h is a light hadronic system with orbital angular momentum L with respect to H'_Q in a state of *total* angular momentum J_h, where $\vec{J}_h = \vec{L} + \vec{S}_h$ and $\vec{S}_h$ is the spin of h. The heavy quark acts as a static source about which the reaction $s_\ell^{\pi_\ell} \to s'_\ell{}^{\pi'_\ell} + h$ occurs. Since the spin of the heavy quark decouples from the light degrees of freedom, it is clear that each such allowed partial wave amplitude of the light degrees of freedom will determine the amplitudes for the four hadronic level processes $s_+^{\pi_\ell} \to s'_+{}^{\pi'_\ell}$, $s_+^{\pi_\ell} \to s'_-{}^{\pi'_\ell}$, $s_-^{\pi_\ell} \to s'_+{}^{\pi'_\ell}$, and $s_-^{\pi_\ell} \to s'_-{}^{\pi'_\ell}$. The amplitudes for these transitions will be of the form

$$
\begin{aligned}
A(H_Q \to [H'_Q h]_{LJ_h}) \sim \sum_{m_\ell, m'_\ell} & C(J_h, m_\ell - m'_\ell; s', m_s + m'_\ell - m_\ell | s, m_s)^* \\
& C(s'_\ell, m'_\ell; s_Q, m_s - m_\ell | s', m_s + m'_\ell - m_\ell)^* \\
& C(J_h, m_\ell - m'_\ell; s'_\ell, m'_\ell | s_\ell, m_\ell) \\
& C(s_\ell, m_\ell; s_Q, m_s - m_\ell | s, m_s)
\end{aligned} \tag{1}
$$

where (s, m_s) and (s', m'_s) are the $\vec{S}^2$ and S_z quantum numbers of H_Q and H'_Q and the C's are Clebsch-Gordan coefficients. Note that although this amplitude may depend on L via a reduced matrix element, the Clebsch-Gordan sum depends only on J_h. From this formula one can directly prove the properties of the decays $s_\pm \to s'_\pm$ described above.

The heavy quark symmetry cannot, of course, tell us anything about the spectroscopy of the light degrees of freedom: it can only predict relationships between heavy quark systems involving given states of these degrees of freedom.

For mesons with flavor quantum numbers $Q\bar{q}$, where q is a light quark, both the constituent quark model and experiment tell us that the ground states have $s_\ell^{\pi_\ell} = \frac{1}{2}^-$, giving $s_-^\pi = 0^-$ and $s_+^\pi = 1^-$ states which we denote by P_Q and V_Q, respectively. The quark model also tells us that the lowest-lying excited states are likely to be those which correspond to having a spin-$\frac{1}{2}$ constituent quark in an $\ell = 1$ state, giving $s_\ell^{\pi_\ell} = \frac{1}{2}^+$ and $\frac{3}{2}^+$ multiplets. At higher masses one expects to encounter additional negative parity states corrsponding to both $\ell = 2$ and to radial excitations.

The quark model and experiment also tell us that the ground state isospin zero baryons with Qud flavor quantum numbers will have $s_\ell^{\pi_\ell} = 0^+$, while the analogous $I = 1$ ground states will have $s_\ell^{\pi_\ell} = 1^+$. The former gives a single $s^\pi = \frac{1}{2}^+$ state which we simply denote by Λ_Q, while the latter quantum numbers lead to degenerate $s_-^\pi = \frac{1}{2}^+$ and $s_+^\pi = \frac{3}{2}^+$ states which we denote by Σ_Q and Σ_Q^*, respectively. The quark model also predicts that the lowest-lying $I = 0$ excited states of Λ_Q will be seven negative parity levels (three $J^P = \frac{1}{2}^-$ states, three $J^P = \frac{3}{2}^-$ states, and one $J^P = \frac{5}{2}^-$ state) and that the lowest-lying $I = 1$ excited states over the Σ_Q and Σ_Q^* will also be seven negative parity states (once again with three $J^P = \frac{1}{2}^-$ states, three $J^P = \frac{3}{2}^-$ states, and one $J^P = \frac{5}{2}^-$ state). In the constituent quark model, these states arise from exciting one of the two light quarks into an $\ell = 1$ state. When combined with the two available light quark spins, one can obtain in this way four $s_\ell^{\pi_\ell}$ multiplets: two with $s_\ell^{\pi_\ell} = 1^-$ and two others with $s_\ell^{\pi_\ell} = 0^-$ and 2^-. The enumeration of $I = 0$ and $I = 1$ states given above follows from considering the symmetries under light quark interchange of the states that can be formed from the separate excitations of each of the two light constituent quarks[17)]. At higher masses one expects to encounter further positive parity states.

The constraints of heavy quark symmetry apply to all possible strong transitions of these states amongst themselves. However, the simplest decays to detect experimentally will probably be those involving the emission of a pion in transition to one of the ground states.

For such transitions, the cumbersome formula (1) enormously simplifies. The resulting decay amplitudes are given in Tables I-IV. (In these Tables, the final state has been given a single subscript since $J_h = L$ for pion emission. In addition, the reduced amplitudes $\alpha^{(n_\ell)}, \beta^{(n_\ell)}, \ldots$ of the Tables are given a superscript (n_ℓ) to denote that they depend not only on the spin s_ℓ, but also on the other quantum numbers needed to uniquely specify the initial state of the light degrees of freedom.) One notes that these amplitudes have all of the properties described above. These Tables can actually easily be used for the general decay $H_Q \to [H'_Q h]_{LJ_h}$ of Eq. (1). We first observe that analogous tables apply for the emission of any pseudoscalar system, and that the same tables, with $s_\ell^{\pi_\ell} \leftrightarrow s_\ell^{-\pi_\ell}$, apply for the emission of scalar systems. Next we note that the emission of h with spin and intrinsic parity $s_h^{\pi_h}$ and orbital angular momentum L creates a state with parity $(-1)^L \pi_h$. It therefore has the same angular momentum and parity quantum numbers as either a pseudoscalar or scalar meson with $L = J_h$, and by Eq. (1) the same Clebsch-Gordan sum.

As an application of these Tables, consider the $s_\ell^{\pi_\ell} = \frac{1}{2}^+$ and $\frac{3}{2}^+$ multiplets expected to constitute the lowest-lying excitations above the $s_\ell^{\pi_\ell} = \frac{1}{2}^-$ states P_Q and V_Q. From Table I we see that the $s_+^\pi = 2^+$ state of the $s_\ell^{\pi_\ell} = \frac{3}{2}^+$ multiplet has decay amplitudes in the proportions $\sqrt{\frac{2}{5}} : \sqrt{\frac{3}{5}}$ to the states $[P_Q\pi]_D$ and $[V_Q\pi]_D$, while its multiplet partner with $s_-^\pi = 1^+$ decays *at the same total rate* exclusively to $[V_Q\pi]_D$. Note that this latter state does not decay to $[V_Q\pi]_S$, even though this is an allowed channel. In fact, from Table II we see that the $s_+^\pi = 1^+$ state of the $s_\ell^{\pi_\ell} = \frac{1}{2}^+$ multiplet decays exclusively to $[V_Q\pi]_S$ and does not decay to $[V_Q\pi]_D$; its $s_-^\pi = 0^+$ multiplet partner decays to $[P_Q\pi]_S$ with the same total decay rate. These results were noted earlier by Rosner[18], who obtained them by taking the $m_Q \to \infty$ limit of a nonrelativistic quark model calculation. The heavy quark symmetry allows us to see that these results are model-independent consequences of QCD in that limit. These predictions are not inconsistent with the existing experimental information[15]. If one interprets the two confirmed states $D_2^*(2460)$ and $D_1(2420)$ as members of the $s_\ell^{\pi_\ell} = \frac{3}{2}^-$ multiplet, then their mass difference is consistent with being a Λ_{QCD}/m_Q correction to the limiting theory. Moreover, the $D_2^* \to D\pi$ and $D_2^* \to D^*\pi$ decays have matrix elements which are in the ratio $0.8 \pm 0.1 \simeq \sqrt{\frac{2}{3}}$. (Here and in what follows we will always quote amplitudes with a phase space and "typical"[19] barrier penetration factor $[p_\pi^{2L+1} exp(-p_\pi^2/1GeV^2)]^{\frac{1}{2}}$ removed.) The ratio of the $D_1 \to D^*\pi$ and $D_2^* \to D^*\pi$ decay amplitudes is 2.3 ± 0.6; especially considering that the D_1 decay may be contaminated by an S-wave admixture from $1/m_Q$ corrections, agreement with the heavy quark symmetry limit again seems satisfactory.

(In quark model calculations[19]), the S-wave decays are predicted to be very strong; this may account for the difficulty of seeing the $s_\ell^{\pi_\ell} = \frac{1}{2}^+$ multiplet expected in this mass range.)

Information on charmed baryons is at this time even more sparse than that on charmed mesons. The symmetry predicts that the ratio of the amplitudes for $\Sigma_c \to \Lambda_c\pi$ and $\Sigma_c^* \to \Lambda_c\pi$ will be unity, but this prediction must await the discovery of the Σ_c^*. More interesting are the many predictions for the decays of the expected negative parity excited states. For example, the $s_+^\pi = \frac{5}{2}^-$ and $s_-^\pi = \frac{3}{2}^-$ Λ_Q states of the $s_\ell^{\pi_\ell} = 2^-$ multiplet are predicted to decay purely by D-waves to $\Sigma_Q\pi$ and $\Sigma_Q^*\pi$ with amplitudes in the ratios $A(\Lambda_Q\frac{5}{2}^- \to \Sigma_Q\pi) : A(\Lambda_Q\frac{5}{2}^- \to \Sigma_Q^*\pi) : A(\Lambda_Q\frac{3}{2}^- \to \Sigma_Q\pi) : A(\Lambda_Q\frac{3}{2}^- \to \Sigma_Q^*\pi) = \sqrt{\frac{2}{9}} : \sqrt{\frac{7}{9}} : \sqrt{\frac{1}{2}} : \sqrt{\frac{1}{2}}$. With Tables I-IV and Eq. (1), similar predictions are available for all strong decays of hadrons containing a single heavy quark.

In the future, much more detailed information on excited charmed mesons and baryons will be available from the study of the weak decays of the large numbers of B's and Λ_b's produced at e^+e^- B factories and high energy hadron colliders. The spectroscopy and branching ratios that follow from heavy quark symmetry can be expected to play an important role in determining the J^P quantum numbers of these heavy states, much as in the past isospin symmetry played an important role in determining the flavor quantum numbers of light hadrons.

References

1. N. Isgur and M.B. Wise, Phys. Lett. **B232**, 113 (1989); Phys. Lett. **B237**, 527 (1990).
2. H. Georgi, Phys. Lett. **B240**, 447 (1990).
3. E. Eichten and B. Hill, Phys. Lett. **B234**, 511 (1990).
4. B. Grinstein, Nucl. Phys. **B339**, 253 (1990).
5. E. Eichten in "Field Theory on the Lattice", Proc. Int. Symp. (Seillac, France, 1987) ed. A. Billoire *et al.*, Nucl. Phys. **B (Proc. Suppl.) 4**, 170 (1988).
6. G.P. Lepage and B.A. Thacker, in "Field Theory on the Lattice", Proc. Intern. Symp. (Seillac, France, 1987) ed. A. Billoire *et al.*, Nucl. Phys. **B (Proc. Suppl.) 4**, 199 (1988).
7. M.B. Voloshin and M.A. Shifman, Yad. Fiz. **45**, 463 (1987) [Sov. J. Nucl. Phys. **45**, 292 (1987)].
8. H.D. Politzer and M.B. Wise, Phys. Lett. **B206**, 681 (1988); Phys. Lett. **B208**, 504 (1988).
9. A.F. Falk, H. Georgi, B. Grinstein and M.B. Wise, Nucl. Phys. **B343**, 1 (1990).
10. S. Nussinov and W. Wetzel, Phys. Rev. **D36, 130** (1987); M.B. Voloshin and M.A. Shifman, Sov. J. Nucl. Phys. **47**, 511 (1988).
11. J.D. Bjorken, invited talk at les Rencontre de Physique de la Vallee D'Aoste, La Thuile, Italy. SLAC-PUB-5278 (1990).
12. N. Isgur and M.B. Wise, CEBAF-TH-90-03 (1990), unpublished; N. Isgur, M. Wise, and M. Youssefmir, CEBAF-TH-90-05 (1990), unpublished.
13. N. Isgur and M.B. Wise, Nucl. Phys. **B348**, 276 (1991).
14. H. Georgi, Nucl. Phys. **B348**, 293 (1991).
15. The Particle Data Group, Phys. Lett. **B239**, 1 (1990).
16. K. Han *et al.* (The CUSB Collaboration), Phys. Rev. Lett. **55**, 36 (1985).
17. See, *e.g.*, N. Isgur in "The New Aspects of Subnuclear Physics," ed. A. Zichichi (Plenum, New York, 1980), p. 107 and references therein.
18. J. Rosner, Comm. Nucl. Part. Phys. **16**, 109 (1986).
19. See, *e.g.*, R. Kokoski and N. Isgur, Phys. Rev. **D35**, 907 (1987).

Table I: strong decays of multiplets with $s_\ell^{\pi_\ell} = \frac{1}{2}^-, \frac{3}{2}^+, \frac{5}{2}^-, \ldots$

s^π	final state	amplitude
$(s_\ell + \frac{1}{2})^{\pi_\ell}$	$[P_Q\pi]_{L=s_\ell+\frac{1}{2}}$	$\sqrt{\frac{s_\ell+\frac{1}{2}}{2s_\ell+2}}\alpha^{(n_\ell)}$
	$[V_Q\pi]_{L=s_\ell+\frac{1}{2}}$	$\sqrt{\frac{s_\ell+\frac{3}{2}}{2s_\ell+2}}\alpha^{(n_\ell)}$
$(s_\ell - \frac{1}{2})^{\pi_\ell}$	$[V_Q\pi]_{L=s_\ell-\frac{3}{2}}$	0
	$[V_Q\pi]_{L=s_\ell+\frac{1}{2}}$	$-\alpha^{(n_\ell)}$

Table II: strong decays of multiplets with $s_\ell^{\pi_\ell} = \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \ldots$

s^π	final state	amplitude
$(s_\ell + \frac{1}{2})^{\pi_\ell}$	$[P_Q\pi]_{L=s_\ell-\frac{1}{2}}$	0
	$[V_Q\pi]_{L=s_\ell-\frac{1}{2}}$	$\beta^{(n_\ell)}$
	$[V_Q\pi]_{L=s_\ell+\frac{3}{2}}$	0
$(s_\ell - \frac{1}{2})^{\pi_\ell}$	$[P_Q\pi]_{L=s_\ell-\frac{1}{2}}$	$\sqrt{\frac{2s_\ell+1}{4s_\ell}}\beta^{(n_\ell)}$
	$[V_Q\pi]_{L=s_\ell-\frac{1}{2}}$	$\sqrt{\frac{2s_\ell-1}{4s_\ell}}\beta^{(n_\ell)}$

Table III: strong decays of multiplets with $s_\ell^{\pi_\ell} = 0^+, 1^-, 2^+, \ldots$; for the case $s_\ell^{\pi_\ell} = 0^+$ only the $\frac{1}{2}^+$ state exists and it can only decay to $[\Sigma^*\pi]_P$.

s^π	final state	amplitude
$(s_\ell + \frac{1}{2})^{\pi_\ell}$	$[\Lambda_Q\pi]_{L=s_\ell+1}$	0
	$[\Sigma_Q\pi]_{L=s_\ell+1}$	$\sqrt{\frac{2s_\ell+1}{3(s_\ell+1)}}\delta_+^{(n_\ell)}$
	$[\Sigma_Q^*\pi]_{L=s_\ell-1}$	$\delta_-^{(n_\ell)}$
	$[\Sigma_Q^*\pi]_{L=s_\ell+1}$	$\sqrt{\frac{s_\ell+2}{3(s_\ell+1)}}\delta_+^{(n_\ell)}$
$(s_\ell - \frac{1}{2})^{\pi_\ell}$	$[\Lambda_Q\pi]_{L=s_\ell-1}$	0
	$[\Sigma_Q\pi]_{L=s_\ell-1}$	$\sqrt{\frac{2s_\ell+1}{3s_\ell}}\delta_-^{(n_\ell)}$
	$[\Sigma_Q^*\pi]_{L=s_\ell-1}$	$-\sqrt{\frac{s_\ell-1}{3s_\ell}}\delta_-^{(n_\ell)}$
	$[\Sigma_Q^*\pi]_{L=s_\ell+1}$	$-\delta_+^{(n_\ell)}$

Table IV: strong decays of multiplets with $s_\ell^{\pi_\ell} = 0^-, 1^+, 2^-, \ldots$; for the case $s_\ell^{\pi_\ell} = 0^-$ only the $\frac{1}{2}^-$ state exists and it is forbidden to decay to $\Sigma\pi$ and $\Sigma^*\pi$.

s^π	final state	amplitude
$(s_\ell + \frac{1}{2})^{\pi_\ell}$	$[\Lambda_Q \pi]_{L=s_\ell}$	$\gamma^{(n_\ell)}$
	$[\Sigma_Q \pi]_{L=s_\ell}$	$\sqrt{\frac{s_\ell}{3(s_\ell+1)}}\delta_0^{(n_\ell)}$
	$[\Sigma_Q^* \pi]_{L=s_\ell}$	$\sqrt{\frac{2s_\ell+3}{3(s_\ell+1)}}\delta_0^{(n_\ell)}$
	$[\Sigma_Q^* \pi]_{L=s_\ell+2}$	0
$(s_\ell - \frac{1}{2})^{\pi_\ell}$	$[\Lambda_Q \pi]_{L=s_\ell}$	$-\gamma^{(n_\ell)}$
	$[\Sigma_Q \pi]_{L=s_\ell}$	$\sqrt{\frac{s_\ell+1}{3s_\ell}}\delta_0^{(n_\ell)}$
	$[\Sigma_Q^* \pi]_{L=s_\ell-2}$	0
	$[\Sigma_Q^* \pi]_{L=s_\ell}$	$\sqrt{\frac{2s_\ell-1}{3s_\ell}}\delta_0^{(n_\ell)}$